6.1

Sequence—Not Just Another Glittery Accessory

Arithmetic and Geometric Sequences

LEARNING GOALS

In this lesson, you will:

- Recognize patterns as sequences.
- Determine the next term in a sequence.
- Write explicit and recursive formulas for arithmetic and geometric sequences.
- Use formulas to determine unknown terms of a sequence.

KEY TERMS

- arithmetic sequence
- geometric sequence
- finite sequence
- infinite sequence

"Ostinato" is a musical term that indicates a repeating pattern of notes. A word that you might be familiar with that is related to "ostinato" is "obstinate," meaning "stubborn".

An ostinato is indeed a stubborn pattern. Musicians commonly use ostinati (the plural of ostinato) to underlay a particular feeling they want a certain song to portray. They may also use it to stabilize a variety of pitches to provide uniformity within a song.

A basso ostinato is a type of ostinato that is used to form a harmonic pattern and is repeated throughout a song. Some argue that the basso ostinato should be thought of more as a device than a form of music.

The term "riff" is the modern day ostinato for popular music. A riff is defined as a short series of notes that create a melody within a melody of a song. Unlike an ostinato, a riff does not need to be repeated throughout the whole song.

You may be familiar with Pachelbel's Canon in D, which features one of the most famous repeating patterns of all time.

Just as with ostinati, when dealing with sequences, you look to identify an underlying pattern. You try to identify what it is that is moving the pattern along, so that you may be able to determine what is coming next.

PROBLEM 1 I Spy With My Little Eye A Pattern!

Patterns, both numerical and physical, can be defined as sequences. Recall, a sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects called terms. An **arithmetic sequence** is a sequence of terms in which the difference between any two consecutive terms is a constant. A **geometric sequence** is a sequence of terms in which the ratio between any two consecutive terms is a constant. A sequence that is neither arithmetic or geometric has a pattern, but there is no common difference or ratio.

Sequences can have a fixed number of terms, or they can continue forever. If a sequence terminates it is called a **finite sequence**. If a sequence continues forever it is called an **infinite sequence**.

An ellipsis is 3 periods which means "and so on." Ellipses are used to represent infinite sequences.

1. Lisa and Ray give the next few terms in the sequence: 1, 1, 1, . . .

Lisa

2, 2, 2, 3, 3, 3

The sequence is writing each natural number three times.

Ray

1, 1, 1, 1, 1, 1, . . .

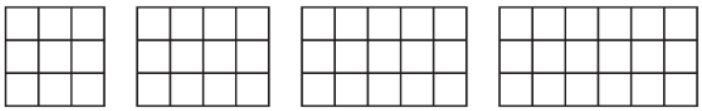
The sequence just repeats 1 forever.

Who is correct? Explain your reasoning.

It is important to recognize that when you are only given the first few terms in a sequence, you may not have enough information to determine the next term.

2. Analyze each sequence and then circle the appropriate type of sequence. If the sequence is arithmetic, identify the common difference. If the sequence is geometric, identify the common ratio. Finally, circle whether the sequence is finite or infinite.

a. number of tiles



Arithmetic Sequence _____ Geometric Sequence _____ Neither

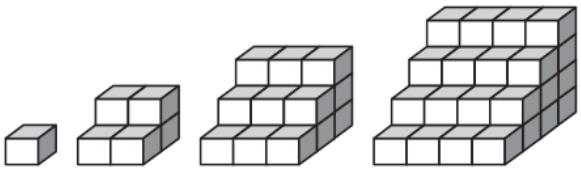
Infinite Sequence Finite Sequence

b. number of toothpicks

Arithmetic Sequence _____ Geometric Sequence _____ Neither

Infinite Sequence Finite Sequence

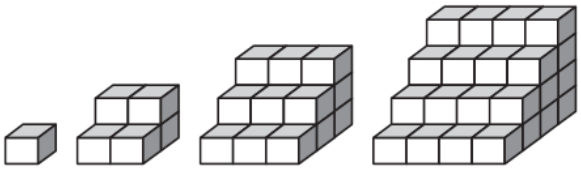
c. number of rows



Arithmetic Sequence _____ Geometric Sequence _____ Neither

Infinite Sequence Finite Sequence

d. sum of the cubes



Arithmetic Sequence _____ Geometric Sequence _____ Neither

Infinite Sequence Finite Sequence

e. number of black triangles

Arithmetic Sequence _____ Geometric Sequence _____ Neither

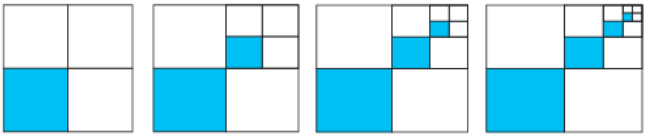
Infinite Sequence Finite Sequence

f. number of white triangles

Arithmetic Sequence _____ Geometric Sequence _____ Neither

Infinite Sequence Finite Sequence

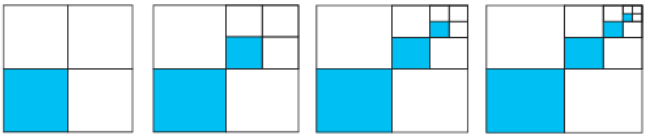
g. Side length of smallest shaded square within the unit square



Arithmetic Sequence _____ Geometric Sequence _____ Neither

Infinite Sequence Finite Sequence

h. Number of shaded squares



Arithmetic Sequence _____ Geometric Sequence _____ Neither

Infinite Sequence Finite Sequence

3. Create your own sequence given the type indicated. Include the first three terms.

a. Arithmetic Sequence

b. Geometric Sequence

c. Neither Arithmetic or Geometric Sequence

PROBLEM 2 Formula: Not Just for Babies

Previously, you learned the explicit and recursive formulas for arithmetic and geometric sequences. An explicit formula for a sequence is a formula used for calculating each term of the sequence using the index, a term's position in the sequence. A recursive formula generates each new term of a sequence based on a preceding term of the sequence.

	Arithmetic Sequence	Geometric Sequence
Explicit Formula	$a_n = a_1 + d(n - 1)$ where a_1 is the first term, d is the common difference, and n is the nth term in the sequence.	$g_n = g_1 \cdot r^{n-1}$ where g_1 is the first term, and r is the common ratio.
Recursive Formula	$a_n = a_{n-1} + d$ where a_{n-1} is the term previous to a_n, and d is the common difference.	$g_n = g_{n-1} \cdot r$ where g_{n-1} is the term previous to g_n, and r is the common ratio.

1. Consider the sequence in Problem 1, Question 1, part (a), *number of tiles*.
   a. Use the recursive formula to determine the 5th term.
   b. Use the explicit formula to determine the 5th term.

2. Consider the sequence in Problem 1, Question 1, part (e), *number of black triangles*.
   a. Use the recursive formula to determine the 5th term.
   b. Use the explicit formula to determine the 5th term.

3. Which formula would you use if you wanted to determine the 95th term of either sequence? Explain your reasoning.

4. Identify each sequence as arithmetic, geometric, or neither. If possible, determine the 50th term of each sequence.

a. $-5, -1, 3, 7, 11, 15, 19, 23 \ldots$

Type of Sequence: _____

50th term: _____

b. $0, 1, 1, 2, 3, 5, 8, 13 \ldots$

Type of Sequence: _____

50th term: _____

c. $27, 9, 3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}$

Type of Sequence: _____

50th term: _____

5. Use either the recursive or explicit formula to determine each answer.

a. The sum of the interior angles in a triangle is 180°, in a quadrilateral is 360°, and in a pentagon is 540°. How many degrees are in a decagon?

b. The employees at Franco's Pizza Shack turn the pizza ovens down to 200° overnight. When the workers open the shop in the morning, they turn the ovens up to 550°. The temperature of each oven increases by 40% every 30 minutes. Will the ovens reach the required 550° in 1.5 hours?

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Talk the Talk

Complete each row in the table using the given information in that row.

	Sequences	Type of Sequence	Recursive Formula	Explicit Formula
A			$g_n = 3(g_{n-1})$ $g_1 = 4$	
B	320, 80, 20, 5, . . .			
C			$a_n = a_{n-1} + 10$ $a_1 = 20$	
D				$g_n = 10 \cdot 5^{n-1}$ $n = 1, 2, 3 \ldots$
E	3, 11, 19, 27 . . .			
F				$a_n = 20(n - 1) + 5$ $n = 1, 2, 3 \ldots$

Be prepared to share your solutions and methods.